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Brief Communication

Transverse galloping at low Reynolds numbers

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Abstract

The possibility of transverse galloping of a square cylinder at low Reynolds numbers ($\mathrm{Re} \leq 200$, so that the flow is presumably laminar) is analysed. Transverse galloping is here considered as a one-degree-of-freedom oscillator subjected to fluid forces, which are described by using the quasi-steady hypothesis (time-averaged data are extracted from previous numerical simulations). Approximate solutions are obtained by means of the method of Krylov-Bogoliubov, with two major conclusions: (i) a square cylinder cannot gallop below a Reynolds number of 159 and (ii) in the range $159 \leq \mathrm{Re} \leq 200$ the response exhibits no hysteresis.

Keywords: Transverse galloping; Laminar flow; Square cylinder

1. Introduction

Among the broad variety of phenomena that flow can induce on structures, transverse galloping is well known to engineers (Simiu and Scanlan, 1978). This is an hydro/aeroelastic instability produced by the interaction of the lateral motion of the elastic body (structure) and the incident flow. Generally, transverse galloping can occur with long elastic bodies of aerodynamically bluff cross-section (non-circular) when the velocity of the incident flow exceeds a certain critical value. Then, the stabilizing effect of structural damping is overcome by the destabilizing effect of the fluid force and an oscillatory motion (normal to the wind flow) develops. This oscillatory motion increases in amplitude until the energy dissipated per cycle by structural damping balances the energy input per cycle from the flow. Sometimes, this amplitude can be many times the characteristic transverse dimension of the structure. Moreover, under certain conditions there is some oscillation hysteresis in the galloping behaviour for a range of flow velocities. This characteristic was observed for the first time by Parkinson (1961, 1964) in the course of laboratory experiments. When hysteresis takes place, multiple solutions for the amplitude of oscillation can appear for a range of flow velocities, depending on whether the flow velocity is increasing or decreasing. Most of the early interest in transverse galloping was directly related to the electrical lines and galloping oscillations sometimes observed when the ice accretion on the wires modified their initially almost circular sections. Thereafter, attention broadened to situations where the phenomenon has also been observed: marine pipelines (Simpson, 1972), traffic signs and signal supports (Johns and Dexter, 1998), gates with underflow (Nguyen and Naudascher, 1986), and some metallic structures (Mahrenholtz and Bardowicks, 1980).

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There is a large body of theoretical and experimental work concerning transverse galloping, much of which is reviewed in Parkinson (1974), Blevins (1990) and Naudascher and Rockwell (1994). For example, large efforts have been devoted to study the galloping features: the influence of the geometry of the cross-section (Novak, 1969, 1972), the influence of the incident turbulence (Novak and Tanaka, 1974), the limits of the quasi-steady hypothesis (Nakamura and Matsukawa, 1987; Hémon and Santi, 2000), or the hysteresis phenomenon (Luo et al., 2003; Barrero-Gil et al., 2009). Those studies are focused in the high Reynolds number (Re) regime and, generally, discarding its effect (many bluff cross-sections have fixed separations points and traditionally the mean flow has been considered, at a first approximation, as Reynolds number independent). However recently, Macdonald and Larose (2006, 2008) have taken into account the Re effect for the case of cable galloping. Near the critical Reynolds number (when the boundary layer upstream of separation changes from laminar to turbulent) a circular cylinder can generate lift. To account for this phenomenon, Macdonald and Larose in their analysis introduced a Re dependence and they showed how a circular cylinder (dry cable) can gallop in a narrow range of Reynolds number (around $270\,000 < \mathrm{Re} < 360\,000$). Nevertheless, the low Reynolds number regime has not received much attention. We believe that this regime may appear in practical situations, for low flow velocities or when the characteristic length scale of the body is small: for example, for an elastic body with a characteristic length of the cross-section of $D = 1\,\mathrm{mm}$, and under the action of an airstream with velocity $U = 1\,\mathrm{m/s}$, the Reynolds number is $\mathrm{Re} = UD/\nu = 100$ (ν is the kinematic viscosity). Based on Sohankar's numerical simulations on the low Reynolds number flow around a square cylinder (Sohankar et al., 1998), the aim of this brief communication is to address two questions:

(i) Can transverse galloping appear at low Reynolds number (laminar regime) for a square section?
(ii) If so, what kind of response exists (whether hysteresis appears or not)?

Following a description of the mathematical modelling of transverse galloping in the next section (Section 2), we use numerical data to study the possibility of transverse galloping and, for those affirmative cases, the body response (Section 3). Finally, some conclusions are drawn.

2. Mathematical modelling of transverse galloping

The description of the behaviour of an elastic body under the action of an incident flow is an extremely complex problem; however, in some cases its modelling can be simplified in order to make an analytical study feasible. Common assumptions are (Parkinson, 1974): (i) the structure is described as a linear oscillator of one-degree-of-freedom (the possibility of rotational motion is not considered), (ii) the structure is sufficiently slender to consider two-dimensional flow, and (iii) that the incident flow is free of turbulence. Under these conditions, the equation governing the dynamics of the transverse galloping represents a balance between inertial, damping, stiffness, and fluid forces (Blevins, 1990):

$$m(\ddot{y} + 2\zeta\omega_y\dot{y} + \omega_y^2 y) = F_y = \tfrac{1}{2}\rho U^2 D C_y, \tag{1}$$

where y denotes the transverse displacement (vertical), m is the body mass per unit length, ζ is the dimensionless structural damping coefficient, ω_y is the undamped natural frequency, ρ is the fluid density, which will be considered constant throughout the analysis, U is the velocity of the incident flow, D is the characteristic dimension of the structure in the direction of the flow (here, D is the side-length of the square cylinder), and C_y is the instantaneous fluid force coefficient in the normal direction to the incident flow; finally, the overdot stands for differentiation with respect to time t.

The fluid force is evaluated by resorting to the quasi-steady assumption, whose use is justified when the following conditions are satisfied:

(i) The characteristic timescale of the body oscillations T_y ($\sim 1/f_y$, where f_y is the natural frequency of oscillations) is much larger than the characteristic timescale of the flow T_R (residence time), of order D/U. Taking as above (Section 1) $U = 1\mathrm{m/s}$, $D = 1\,\mathrm{mm}$, and $f_y = 1\,\mathrm{Hz}$, then a reduced velocity $U_R = U/(f_y D) = T_y/T_R = 1000$ is obtained (high enough to consider quasi-steady conditions).
(ii) The vortex shedding frequency f_t is much higher than the frequency of oscillations. $f_t \sim U\mathrm{St}/D$, where St is the Strouhal number. For a square section, and the Reynolds numbers considered, a representative value of $\mathrm{St} = 0.1$ can be assumed (Okajima, 1982). Then $f_t \sim 100\,\mathrm{Hz} \gg f_y$.

Thus, the fluid force is completely determined by the instantaneous velocity of oscillation of the structure, and fluid force data in the static case can be used and they can be related to the motion of the body.

In the static case, the force coefficient (normal to the incident flow) can be expanded in powers of the angle of attack, α, in the range of interest, $[-\alpha^*, \alpha^*]$; α^* being moderately small,

$$C_y(\alpha) = \sum_{j=0}^{n} a_j \alpha^j, \tag{2}$$

where α is the angle between the incident flow and the reference direction (in the static equilibrium position of the body, see sketch shown in Fig. 2). Assuming small values of the velocity ratio $\dot{y}/U$ and expanding α in Taylor series, $\alpha = \tan^{-1}(\dot{y}/U) \simeq \dot{y}/U$, one obtains at the lowest order

$$C_y(\alpha) = \sum_{j=0}^{n} a_j \left(\frac{\dot{y}}{U}\right)^j. \tag{3}$$

For the present study, the employed polynomial approximation is

$$C_y(\alpha) = a_1\left(\frac{\dot{y}}{U}\right) + a_3\left(\frac{\dot{y}}{U}\right)^3 + a_5\left(\frac{\dot{y}}{U}\right)^5 + a_7\left(\frac{\dot{y}}{U}\right)^7. \tag{4}$$

Note that we consider only odd terms due to the symmetry of the square section. On the other hand, it is common to employ a seventh degree polynomial to approximate $C_y(\alpha)$ (Parkinson, 1974). Substituting Eq. (4) in Eq. (1) and introducing dimensionless variables $\eta = y/D$ and $\tau = \omega_y t$ and the reduced velocity $U_r = U/(\omega_y D)$, one gets

$$\eta'' + 2\zeta\eta' + \eta = \mu U_r^2 \sum_{j=1,3,5,7} a_j \left(\frac{\eta'}{U_r}\right)^j, \tag{5}$$

where the prime represents differentiation with respect to the dimensionless time τ and $\mu = \rho D^2/2m$ is the dimensionless mass ratio.

Eq. (5) can be solved either numerically or by asymptotic methods if the nonlinear term is small. In the case that both aerodynamic and damping forces, of order of μU_r and ζ, respectively, are small compared with inertia and stiffness forces (of the order of unity in the dimensionless equation), solutions of Eq. (5) will tend to a limit cycle of quasi-harmonic oscillations. This behaviour of the structure is quite usual, as the above-mentioned conditions are fulfilled when the mean density of the structure is much higher than that of the fluid (for air μ is typically of order 10^{-3} and $\mu U_r \sim 10^{-2}$) and the value of the structural damping coefficient rarely exceeds 1%. In this case, applying the method of Krylov–Bogoliubov to Eq. (5) (Murdock, 1991), one obtains the evolution of the dimensionless amplitude R ($R = A/D$; A being the amplitude of oscillations) of oscillations (Luo et al., 2003),

$$\frac{dR^2}{d\tau} = \mu a_1\left[\left(U_r - \frac{2\zeta}{\mu a_1}\right)R^2 + \frac{3}{4}\left(\frac{a_3}{a_1 U_r}\right)R^4 + \frac{5}{8}\left(\frac{a_5}{a_1 U_r^3}\right)R^6 + \frac{35}{64}\left(\frac{a_7}{a_1 U_r^5}\right)R^8\right]. \tag{6}$$

Eq. (6) determines the galloping behaviour. The steady oscillation amplitudes are the real and positive solutions of $dR^2/d\tau = 0$. One, two or three solutions are possible. When the solution is not unique, the final amplitude of oscillation depends on whether the flow velocity is increasing or decreasing. This is a phenomenon of hysteresis and its appearance is due to the emergence of inflection points (at least one) in the $C_y(\alpha)$ curve (Barrero-Gil et al., 2009).

The functional dependence of R can be deduced from (6) (as well as by means of simple dimensional considerations): $R = \pi_1(\mu, \zeta, U_r, a_{1,3,5,7})$. Traditionally, in the high Reynolds number regime, a_i are considered constants (at least, as a first approximation), and can be considered that $R = \pi_2(\mu, \zeta, U_r)$. However, in the laminar regime (considered here as $\mathrm{Re} < 200$), coefficients a_i can be a function of the Reynolds number (in this regime it is well known that the flow around the bluff body is strongly dependent on Re (Wu et al., 2006)). Then, Re appears as a new parameter in the problem: $R = \pi_3(\mu, \zeta, U_r, \mathrm{Re})$. In the next section a relationship taking into account the Reynolds number influence is proposed.

3. Galloping at low Reynolds numbers

The values of coefficients a_i ($i = 1, 3, 5, 7$) used here are computed from Sohankar's numerical simulations on the 2-D flow around a square cylinder and are shown in Table 1. The relationship between the polynomial coefficients and the

Table 1
Steady force coefficients computed numerically by Sohankar et al. (1998) and polynomial approximation of $C_y(\alpha)$, as a function of Reynolds number.

	$\alpha[°]$	C_D	C_L	$C_y(= -C_L\cos\alpha - C_D\sin\alpha)$	$C_y(\alpha) = \sum_{1,3,5,7} a_j\alpha^j$
Re = 50	0	1.6	0.000	0.00	$a_1 = -1.8$
	10	1.52	0.035	−0.30	$a_3 = 2.4$
	20	1.50	0.040	−0.55	$a_5 = -7.1$
	30	1.54	0.025	−0.79	$a_7 = 6.8$
Re = 100	0	1.46	0.000	0.00	$a_1 = -1.0$
	5	1.40	−0.025	−0.09	$a_3 = -10.0$
	10	1.41	−0.020	−0.22	$a_5 = 64.0$
	15	1.45	0.010	−0.38	$a_7 = -130.0$
	20	1.51	0.025	−0.54	
	30	1.62	0.035	−0.53	
Re = 150	0	1.42	0.000	0.00	$a_1 = -0.1$
	5	1.35	−0.100	−0.01	$a_3 = -31.0$
	10	1.40	−0.080	−0.16	$a_5 = 190.0$
	15	1.51	−0.025	−0.36	$a_7 = -370.0$
	20	1.62	0.030	−0.58	
	30	1.78	0.050	−0.93	
Re = 200	0	1.44	0.000	0.00	$a_1 = 0.7$
	5	1.36	−0.180	0.07	$a_3 = -55.0$
	10	1.47	−0.120	−0.13	$a_5 = 370.0$
	15	1.64	−0.050	−0.37	$a_7 = -750.0$
	20	1.77	0.020	−0.62	
	30	1.90	0.030	−0.97	

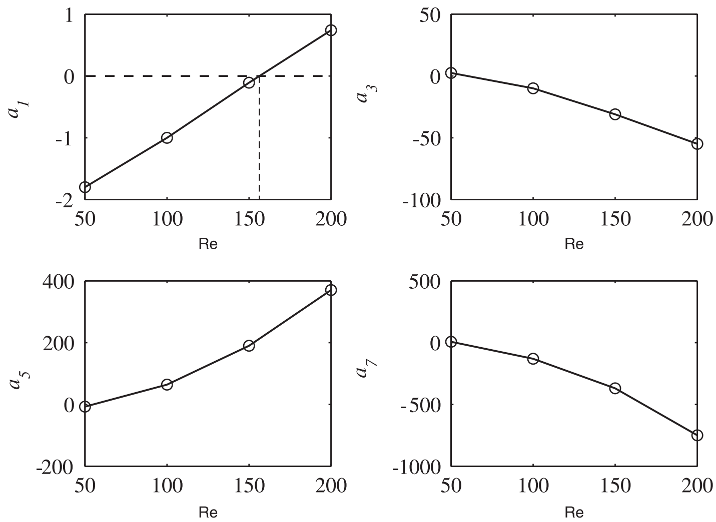

Fig. 1. a_1, a_3, a_5, and a_7 coefficients of polynomial curve fit of $C_y(\alpha)$ as a function of Reynolds number (Re), based on numerical results from Sohankar et al. (1998).

Reynolds number (Re) is shown in Fig. 1, and can be approximated by

$$a_1 = -2.7 + 0.017\mathrm{Re}, \quad a_3 = 10.0 - 0.096\mathrm{Re} - 0.001\mathrm{Re}^2,$$
$$a_5 = -24.0 - 0.210\mathrm{Re} + 0.011\mathrm{Re}^2, \quad a_7 = 13.0 + 1.100\mathrm{Re} - 0.024\mathrm{Re}^2. \tag{7}$$

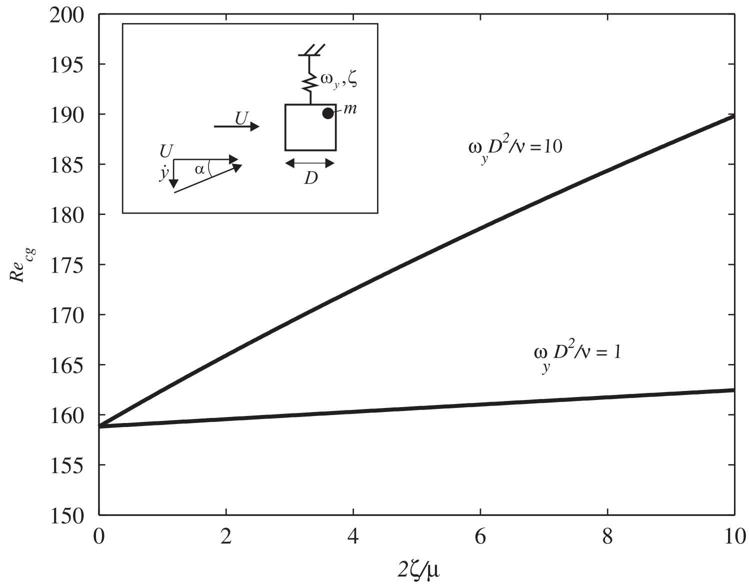

Fig. 2. Critical Reynolds number for the onset of galloping, Re_{cg}, for different damping (ζ), mass ratio (μ), and stiffness levels (ω_y) (see Eq. (7)).

The Reynolds number is directly related to the reduced velocity through $\mathrm{Re} = U_r \omega_y D^2/v$. Then, substituting coefficients given in Eq. (7) in Eq. (6) one gets a functional relationship between the steady amplitude of oscillations, the elastic and the flow properties $[R = \pi(\mu, \zeta, U_r, \omega_y D^2/v)]$.

Conditions for the onset of galloping are reached when the first term of the right-hand side (rhs) in Eq. (6) (linear damping) is equal to zero. Therefore, the critical Reynolds number Re_{cg} for the onset of galloping is given by (note that the expression of a_1 given in Eq. (7) has been introduced in the first term of the rhs in Eq. (6) and that $U_r = v\mathrm{Re}/(\omega_y D^2)$),

$$0.017 \frac{\mathrm{Re}_{cg}^2}{\omega_y D^2/v} - 2.7 \frac{\mathrm{Re}_{cg}}{\omega_y D^2/v} - \frac{2\zeta}{\mu} = 0. \tag{8}$$

Linear stability boundaries (Re_{cg}) are shown in Fig. 2 for different values of elastic properties (μ, ζ), geometrical (D), and incident flow properties (v). It is interesting to note that for extremely low values of the elastic properties there is a lower limit of $\mathrm{Re} = \mathrm{Re}_{cgl} = 159$ for the onset of galloping (when a_1 takes a zero value[1]). Concerning the effect of the oscillatory Reynolds number parameter, $\omega_y D^2/v$, the larger the stiffness of the elastic body, the larger is the Reynolds number at which galloping appears (note that D^2/v is a viscous time scale. This parameter is also known as Stokes number or Roshko number).

Once the relationship between the polynomial coefficients and the Reynolds number has been introduced, Eq. (6) (in fact rhs of Eq. (6) = 0) provides an implicit relationship between the steady dimensionless amplitude of oscillations R^* and U_r, $2\zeta/\mu$, and $\omega_y D^2/v$ (or Re). This relationship is shown in Fig. 3 (obtained with the aid of MATLAB software), where the steady oscillation amplitude versus reduced velocity (and hence Reynolds number) is presented for several values of $2\zeta/\mu$ and $\omega_y D^2/v$. In all cases, there is no hysteresis (only one amplitude corresponds to a given value of U_r) in the range under study (Re < 200). At this point, an explanation is necessary: higher Reynolds numbers has not been considered due to the fact that for Re > 200 the flow is inherently three-dimensional and a 3-D computation is needed (usually it is believed that for Re < 200 the flow is laminar, but two recent studies by Tong et al., 2008 and Sheard et al., 2009, show a 3-D transition at Reynolds numbers of approximately 160 for the square cylinder). When the flow is 3-D, new parameters need to be considered (as the aspect ratio of the square cylinder) and numerical simulations are less reliable. On the other hand, we have not found experimental data in the literature of $C_y(\alpha)$ for a square section for low Reynolds numbers. The only experimental results available in the open literature seem to be for cases where $\alpha = 0°$.

[1]Note that the value of Re_{cgl} is an approximate one and depends on the accuracy of the coefficients given in (7).

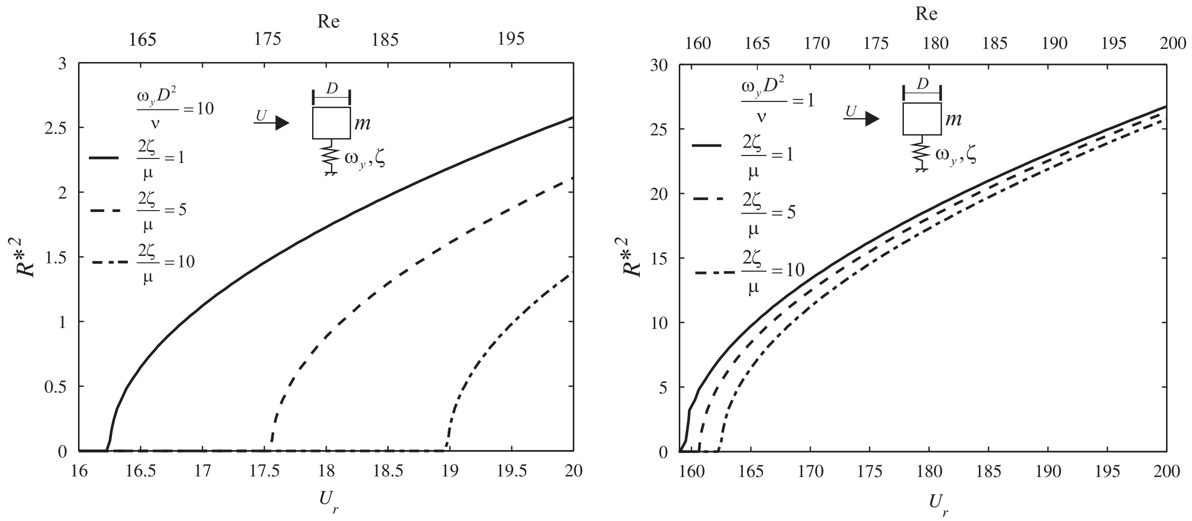

Fig. 3. Galloping response of a square cylinder at low Reynolds number for different damping (ζ), mass ratio (μ), and stiffness levels (ω_y) (predicted by Eq. (6)).

Moreover, wind/water tunnel tests of galloping of square cylinders at low Reynolds numbers do not seem to exist (note the difficulty of performing experiments at low Re due to characteristic dimensions and elastic properties required).

4. Conclusions

In this Brief Communication the phenomenon of transverse galloping at low Reynolds numbers for a square cylinder has been analysed. The analysis, based on numerical simulations, confirms the possibility of galloping for Re > 159. Moreover, in the range under study (159 < Re < 200) the analysis shows that there is no hysteresis in the galloping response: for a certain Re($\geq$ 159), the amplitude of oscillations grows from sensibly zero to steady oscillation of finite amplitude R^* and constant frequency. A closer investigation of the problem, both numerical and experimental, is currently underway, in order to validate the predictions of the analytical models presented in this communication.

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